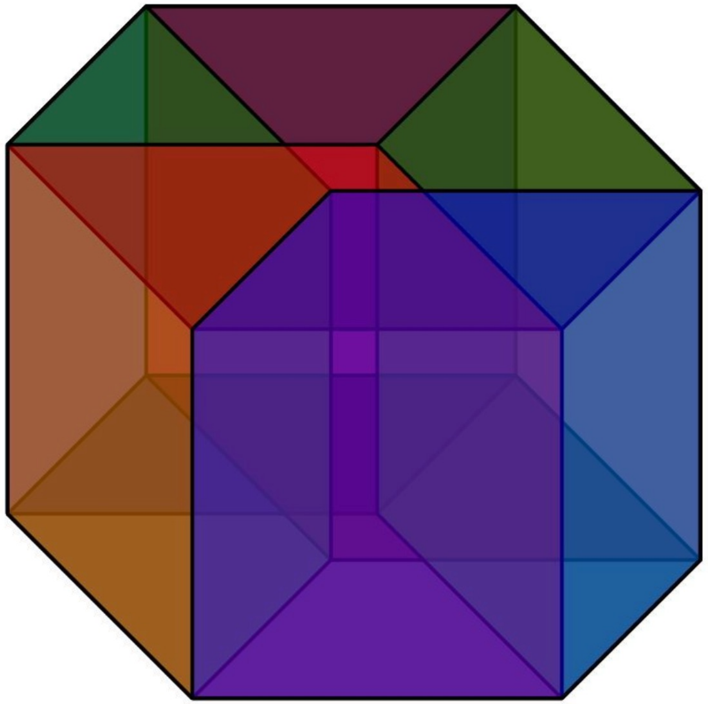


Presenter: Devin Bristow

Mathematical Exploration of Cubes, Spheres, and Polytopes in Several Dimensions

College

Undergraduate

Pepperdine University

B.S. Computer Science

B.S. Mathematics

College

Graduate Research

Brown University

Research Topics:

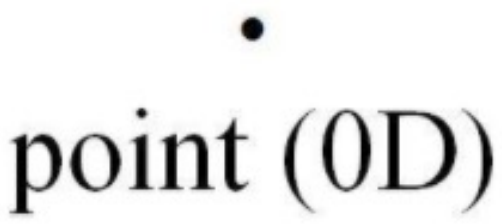
- Topology
- Differential Geometry
- Supersymmetry

Paper: https://ui.adsabs.harvard.edu/abs/2020arXiv201214015B/abstract

Dimensions

Dimension is a measure of extent.

A point is 0 dimensions.



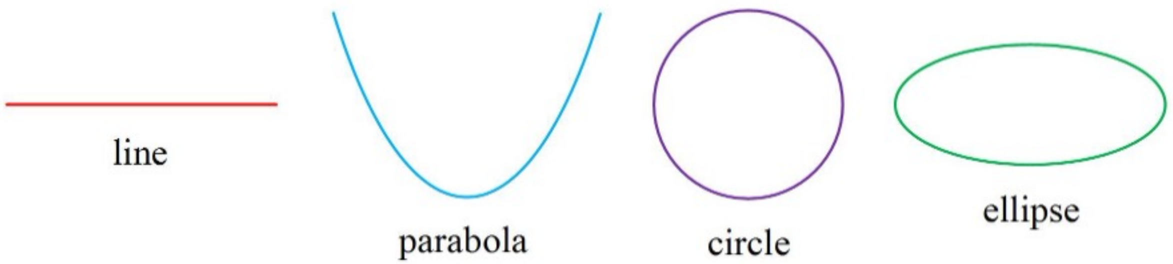
0 dimensions take up an entire space.

It’s the ultimate prison, where you can’t move anywhere.

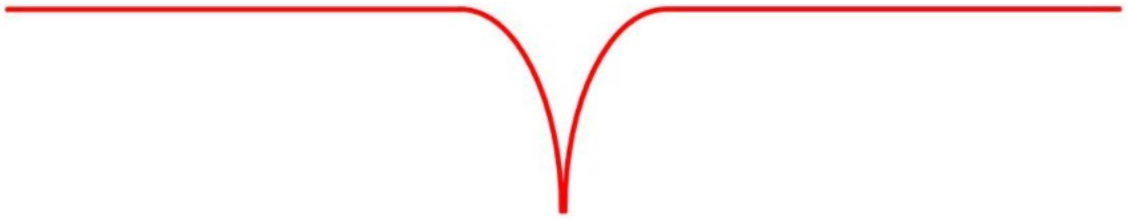
A line is 1 dimension.

line (1D)

1-Dimensional objects



1-Dimensional objects



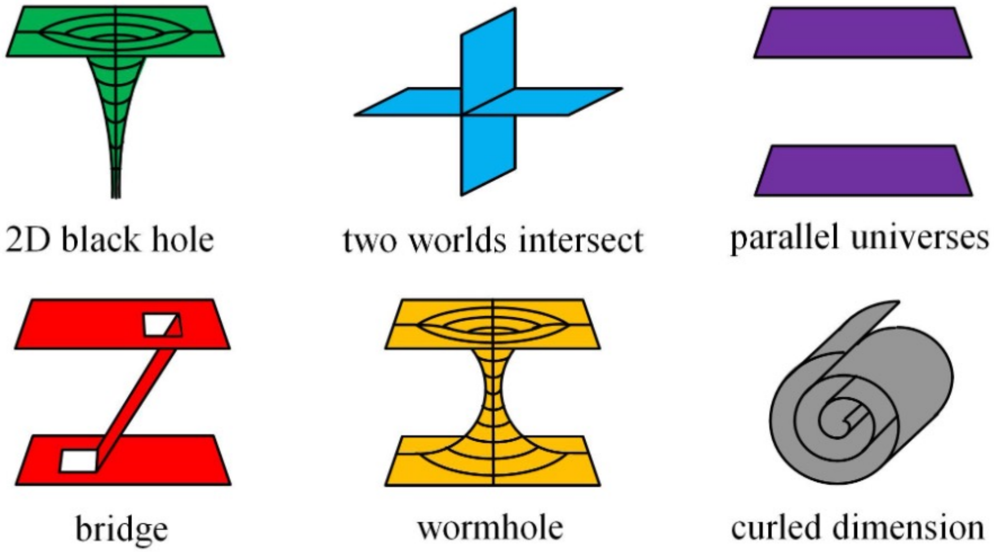
1D black hole

A rectangle is 2 dimensions.

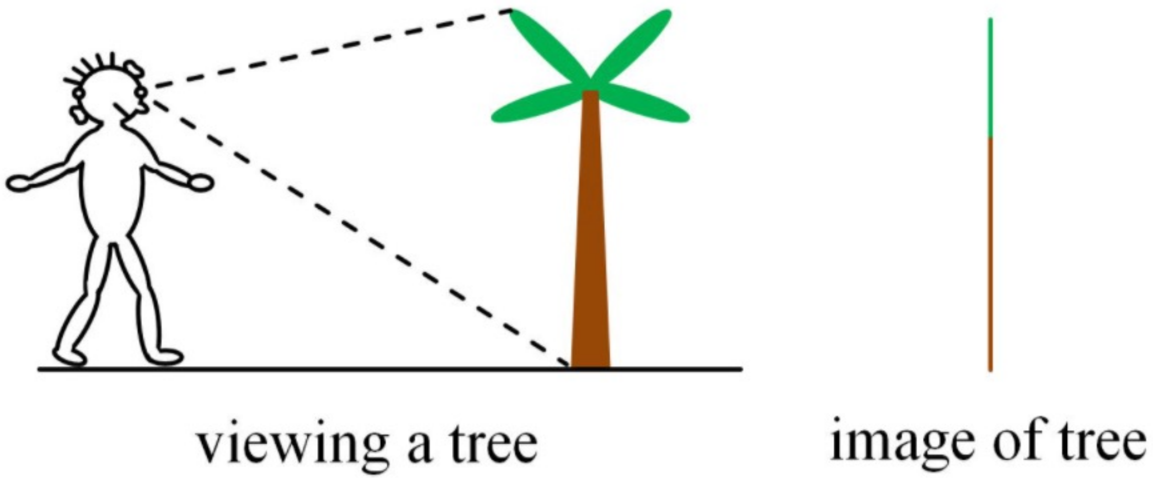
rectangle (2D)

2-Dimensional objects

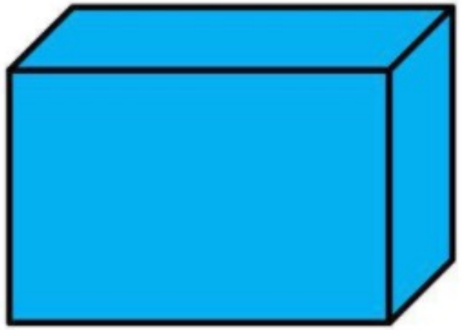
2-Dimensional objects



2-Dimensional world

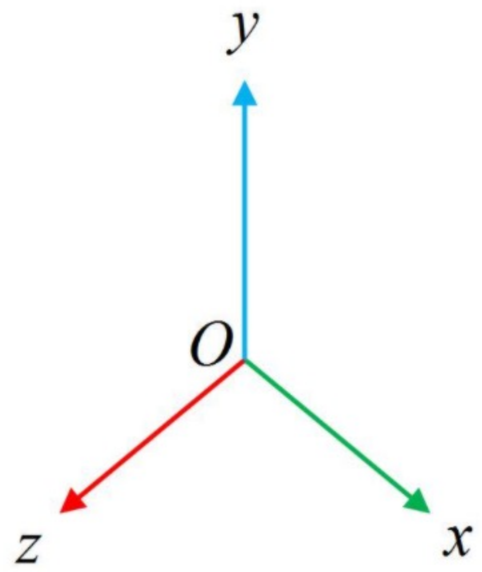


A block is 3 dimensions.

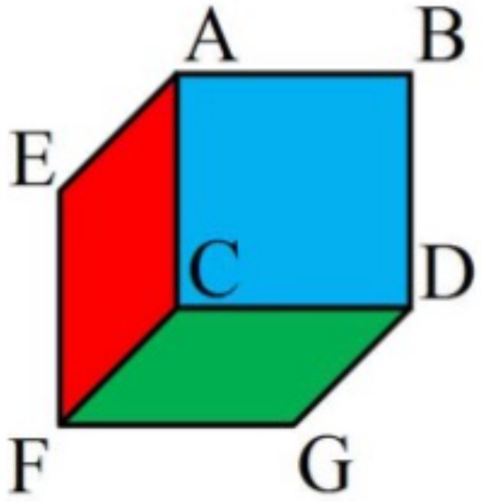


block (3D)

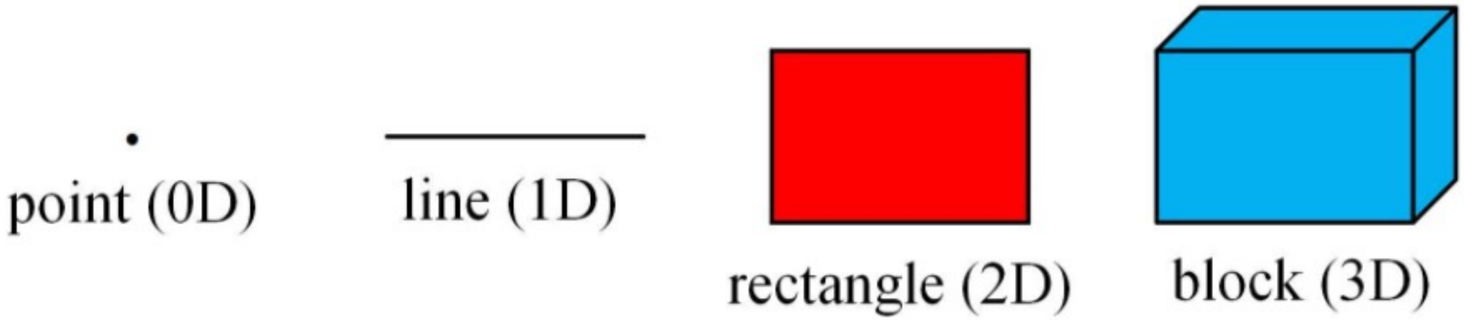
3-Dimensional axis



Corner of a room



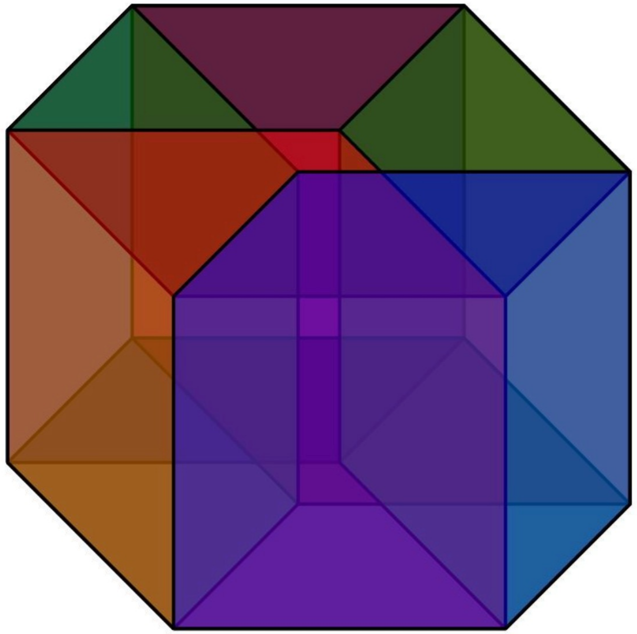
Summary



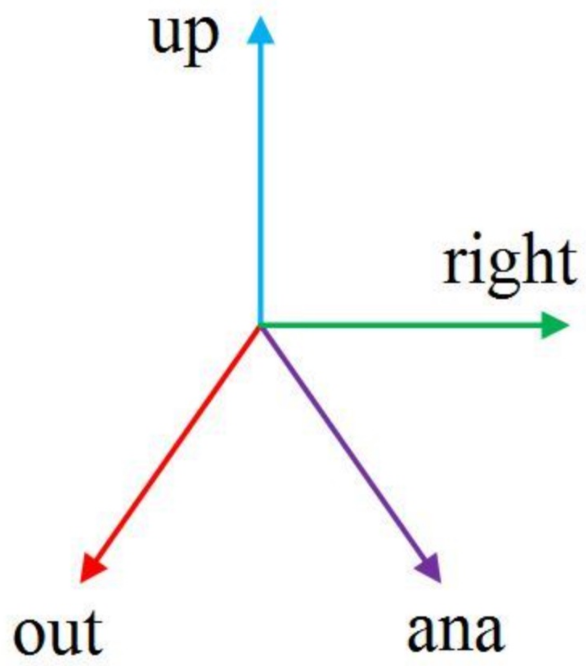
From left to right:

- 0d) a monkey's thoughts
- 1d) a monkey's tail
- 2d) a monkey's shadow
- 3d) a monkey itself

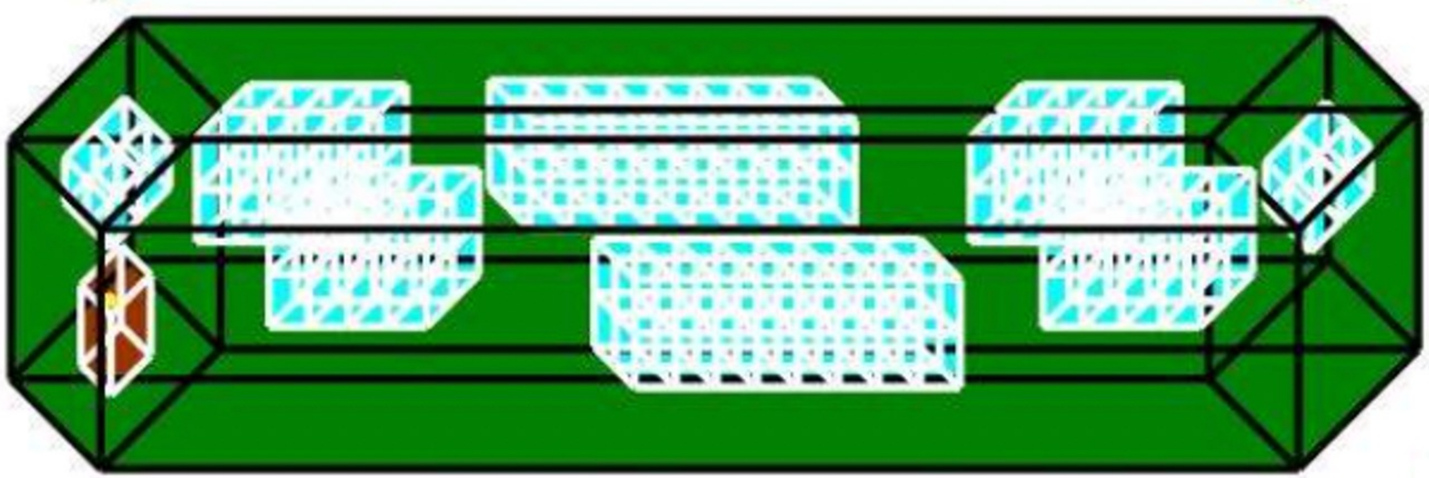
A tesseract is 4 dimensions.



4-Dimensional axis



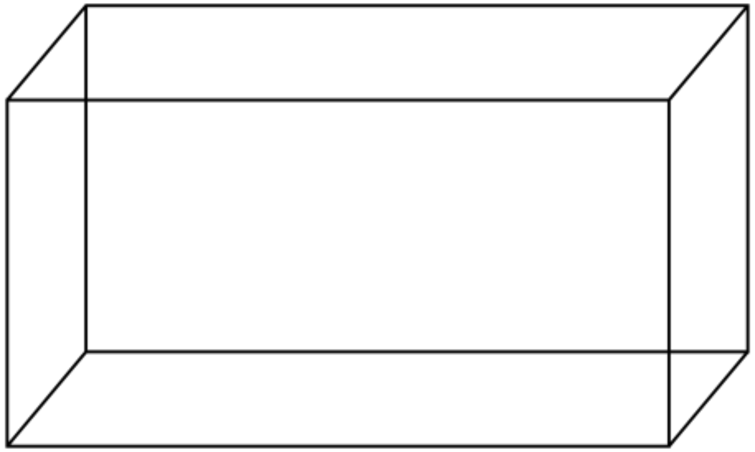
4-Dimensional House



4-Dimensional Room

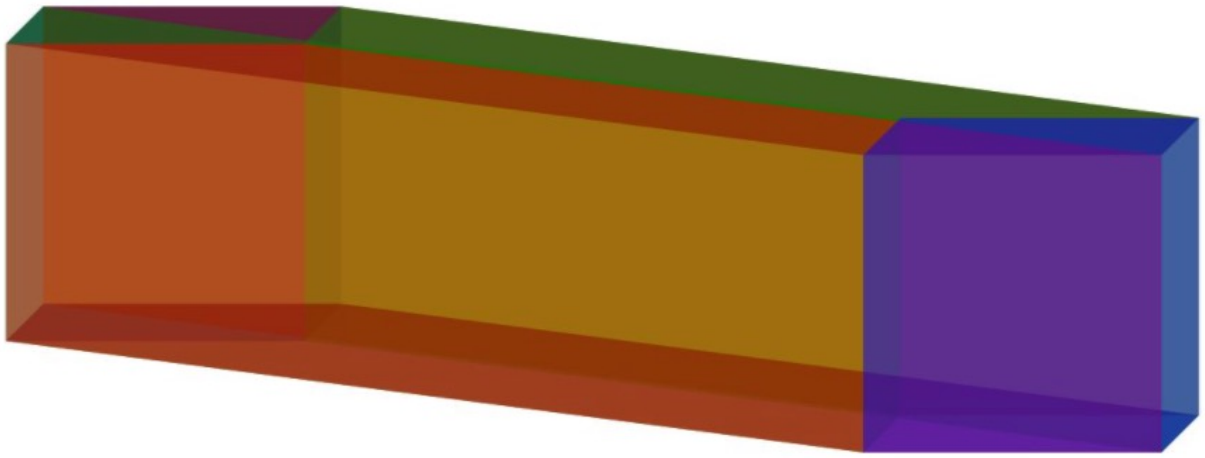
A typical 4D room would be rectangular,
not a square.

Cuboid



2 or more of the 6 sides of the cuboid are rectangles.

Hypercuboid

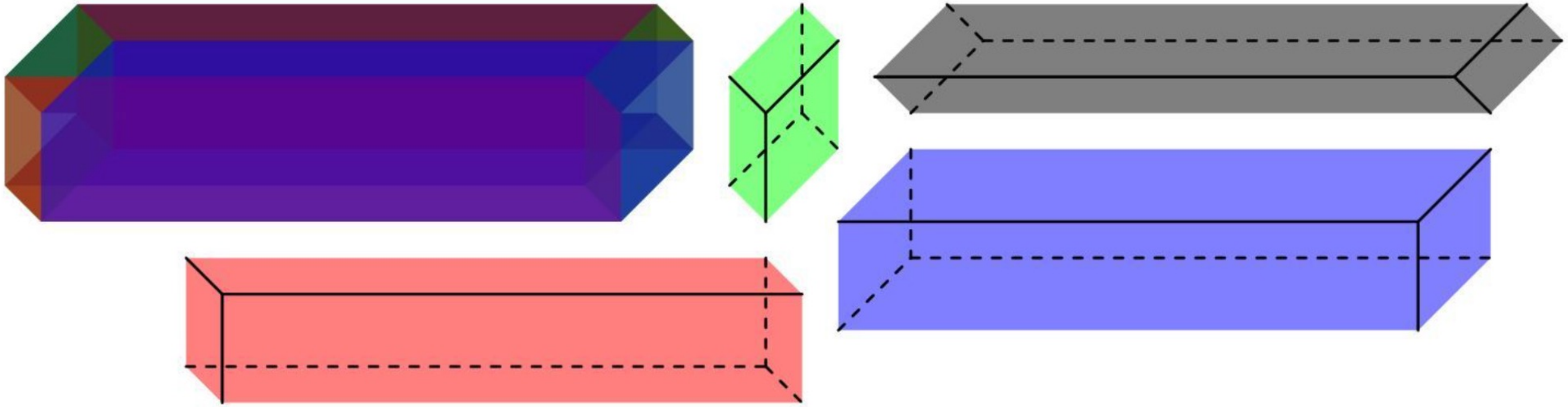


The analogous 4D rectangular hyperbox is called a hypercuboid.

Hypercuboid

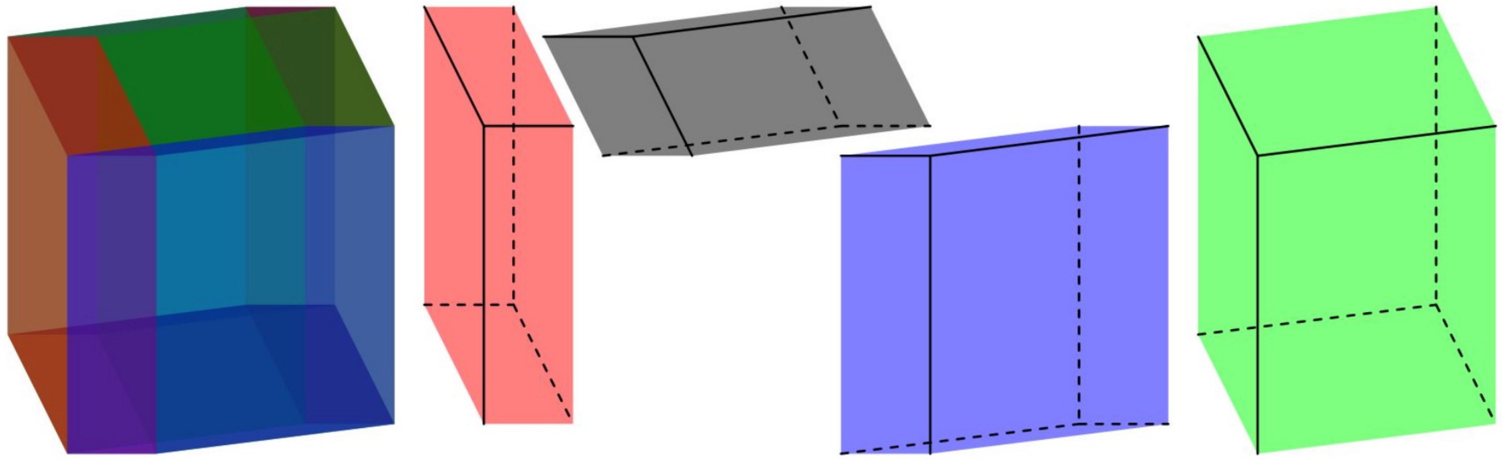
At least 2 of the hypercuboid’s 8 bounding walls are cuboids.

Hypercuboid



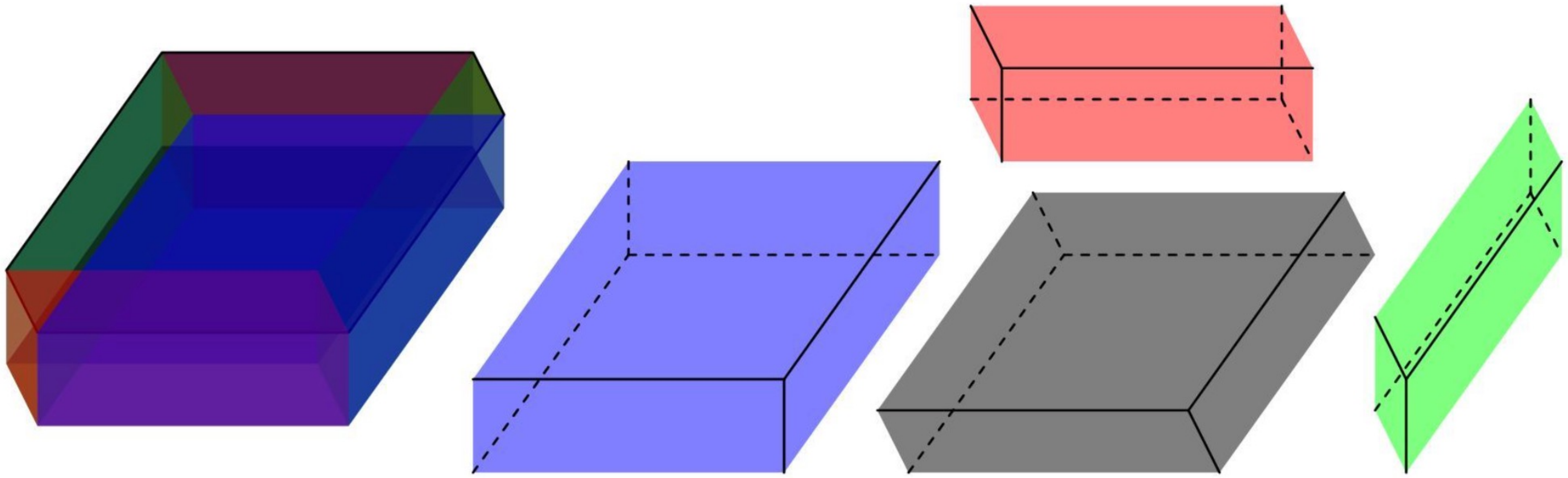
Another example of a hypercuboid
and its bounding cuboids.

Hypercuboid



A hypercuboid with a different set of bounding cuboids.

Hypercuboid

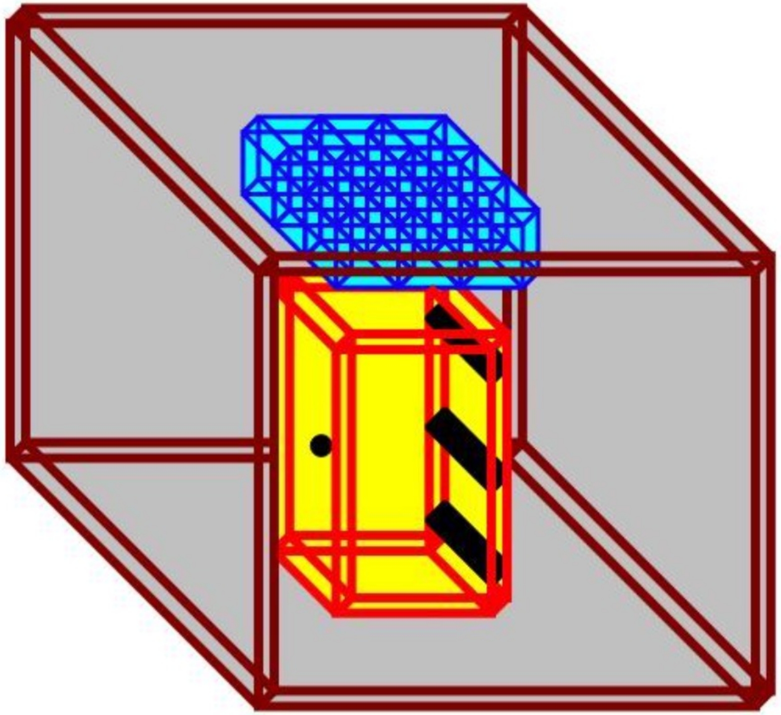


A hypercuboid with yet another
set of bounding cuboids.

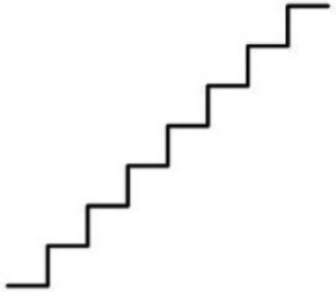
4-Dimensional Room

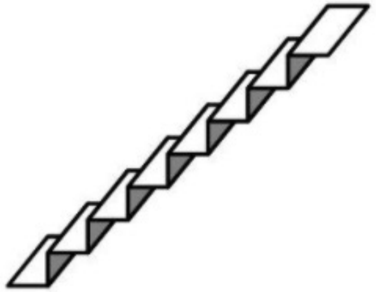
So, a typical 4D room will have the shape of a hypercuboid.

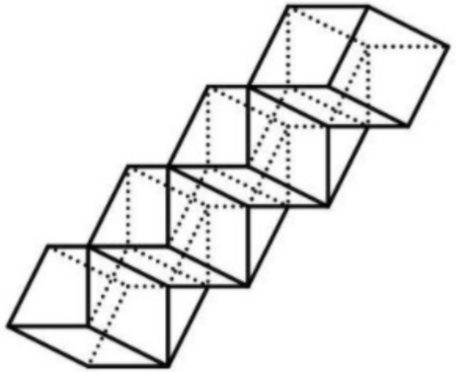
4-Dimensional Wall



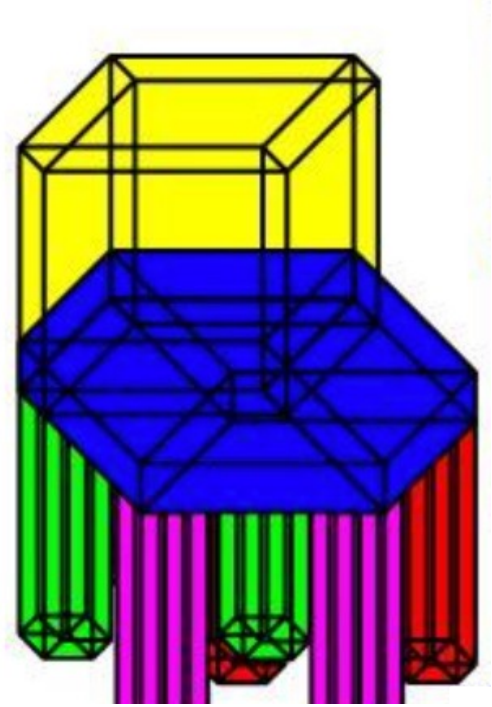
4-Dimensional Stairs


a 2D staircase

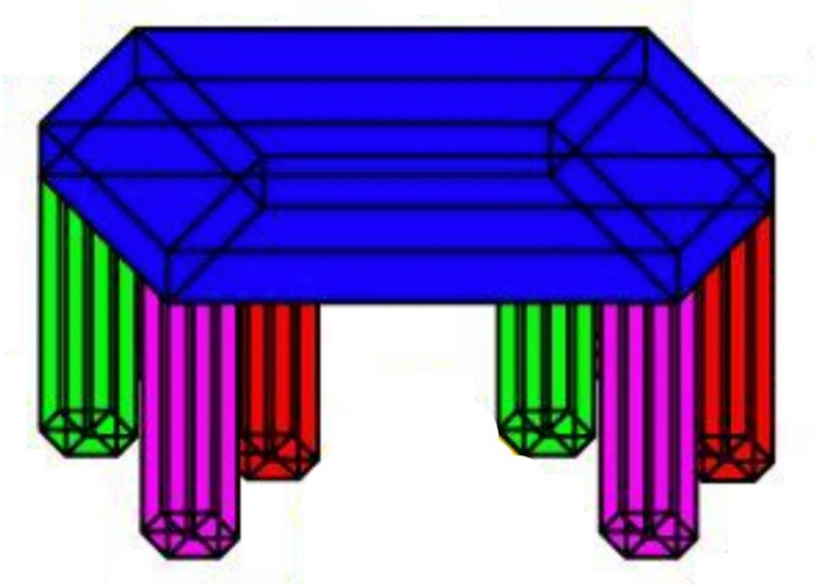

a 3D staircase


a 4D staircase

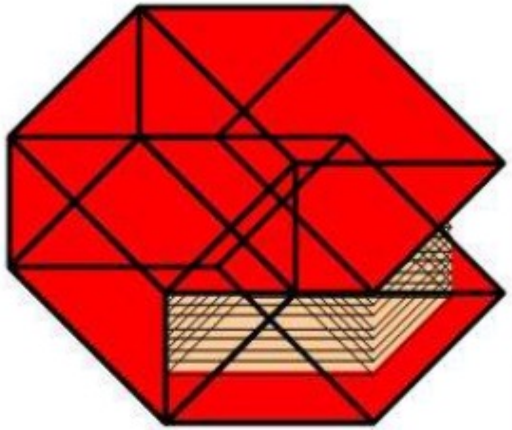
4-Dimensional Chair

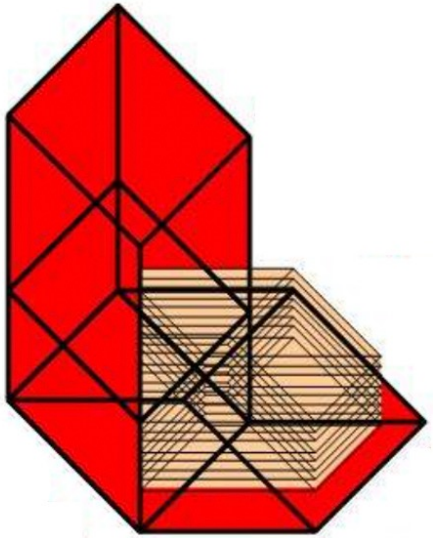


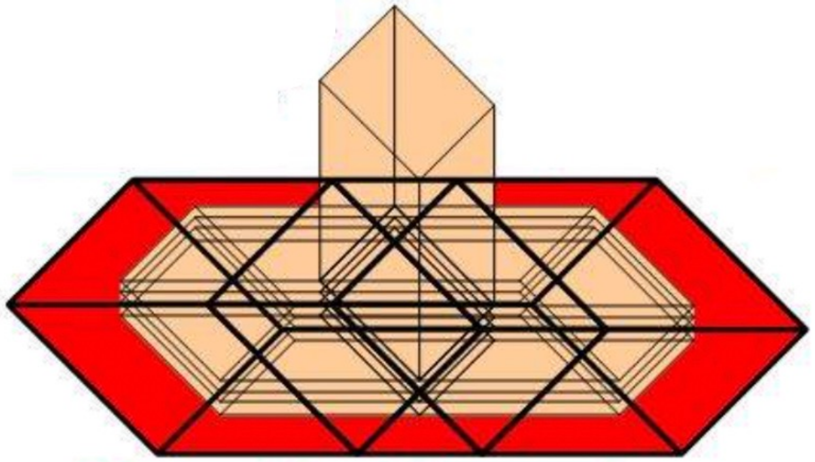
4-Dimensional Table



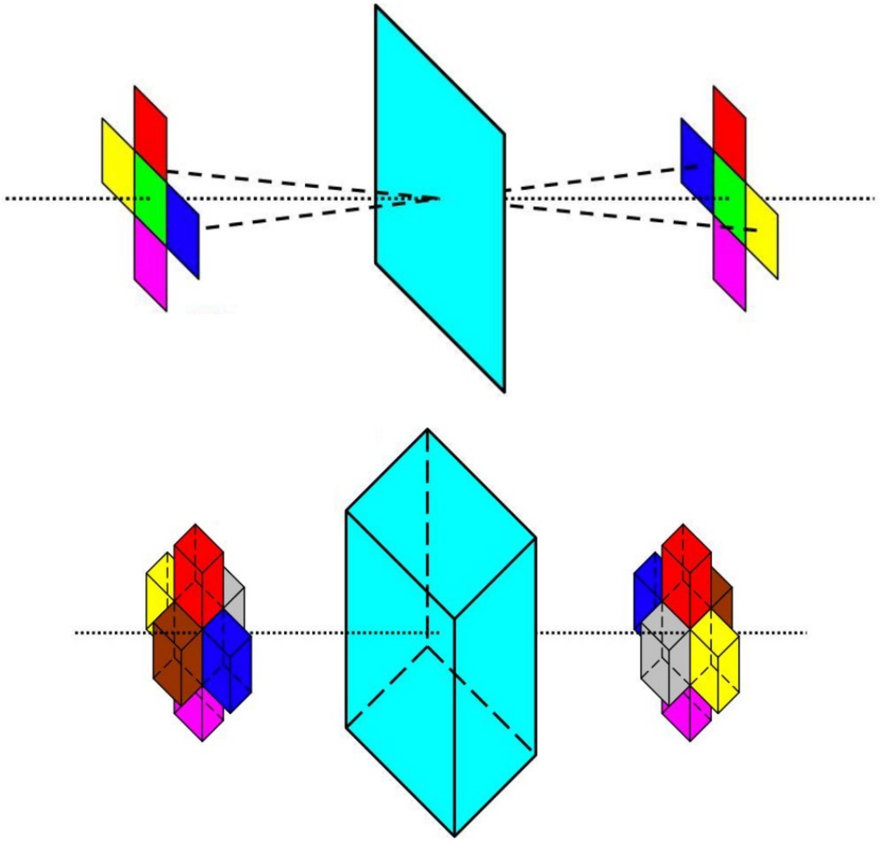
4-Dimensional Book



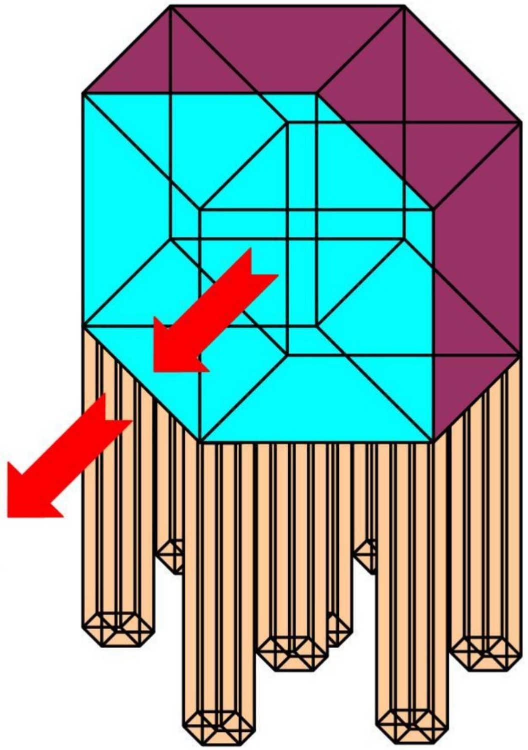




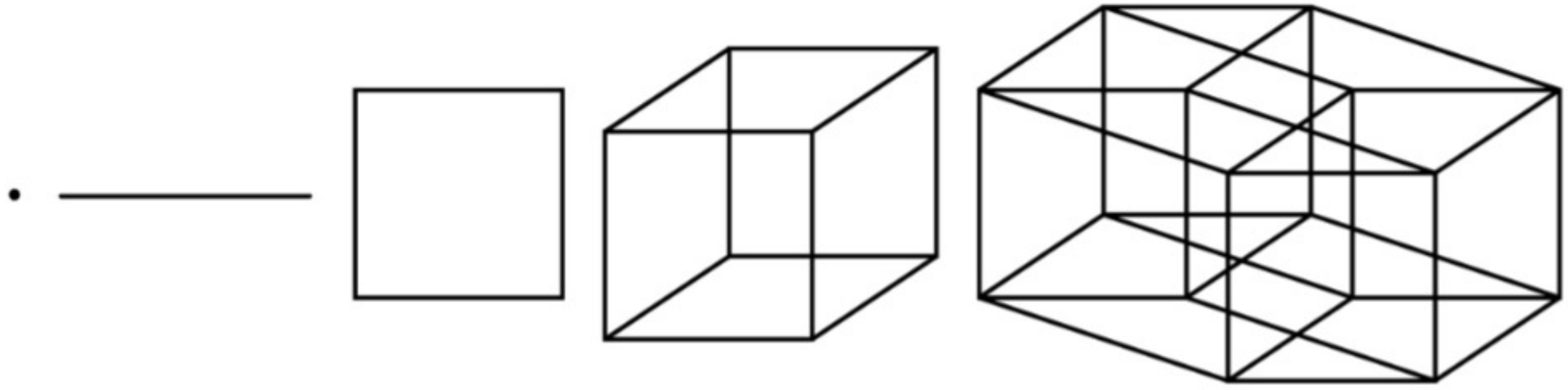
4-Dimensional Mirrors



4-Dimensional TV



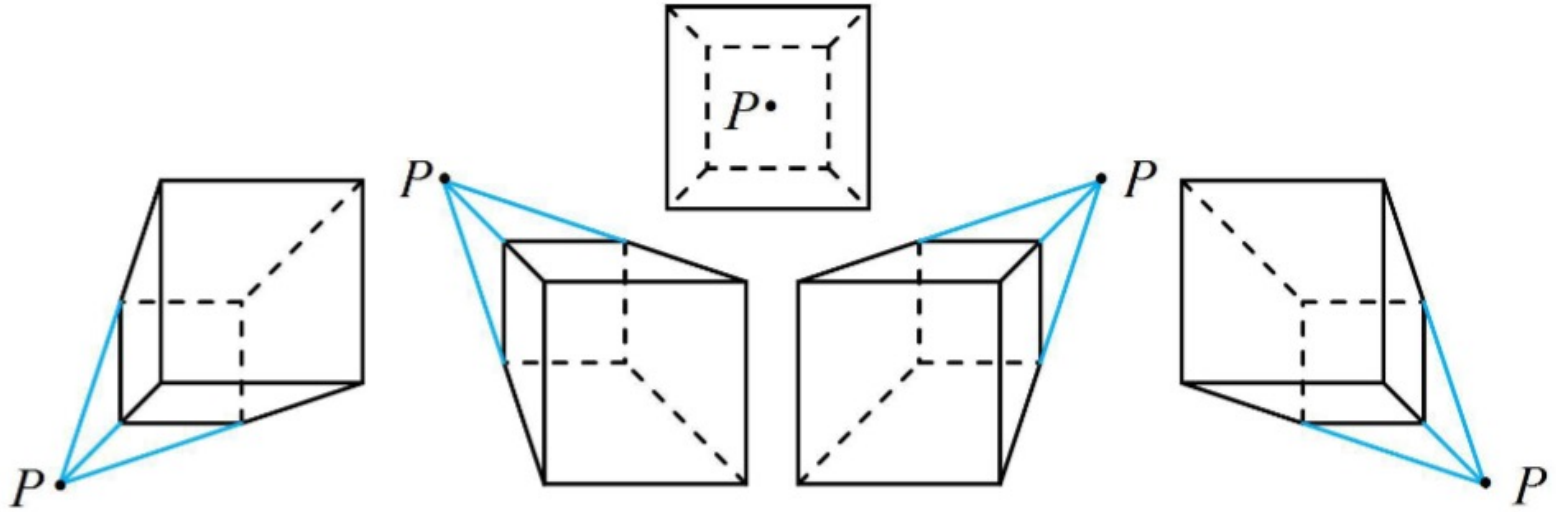
N-Dimensional Cubes



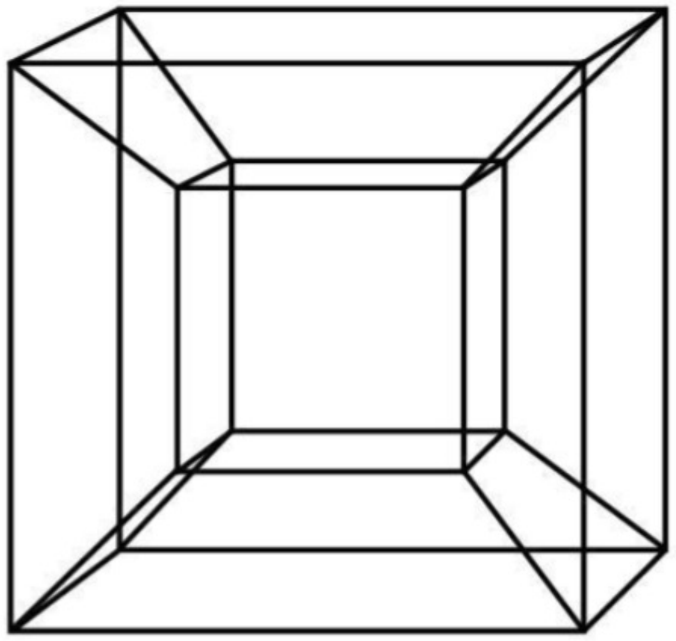
0D 1D 2D 3D 4D

N-dimensional objects in N-dimensional space.

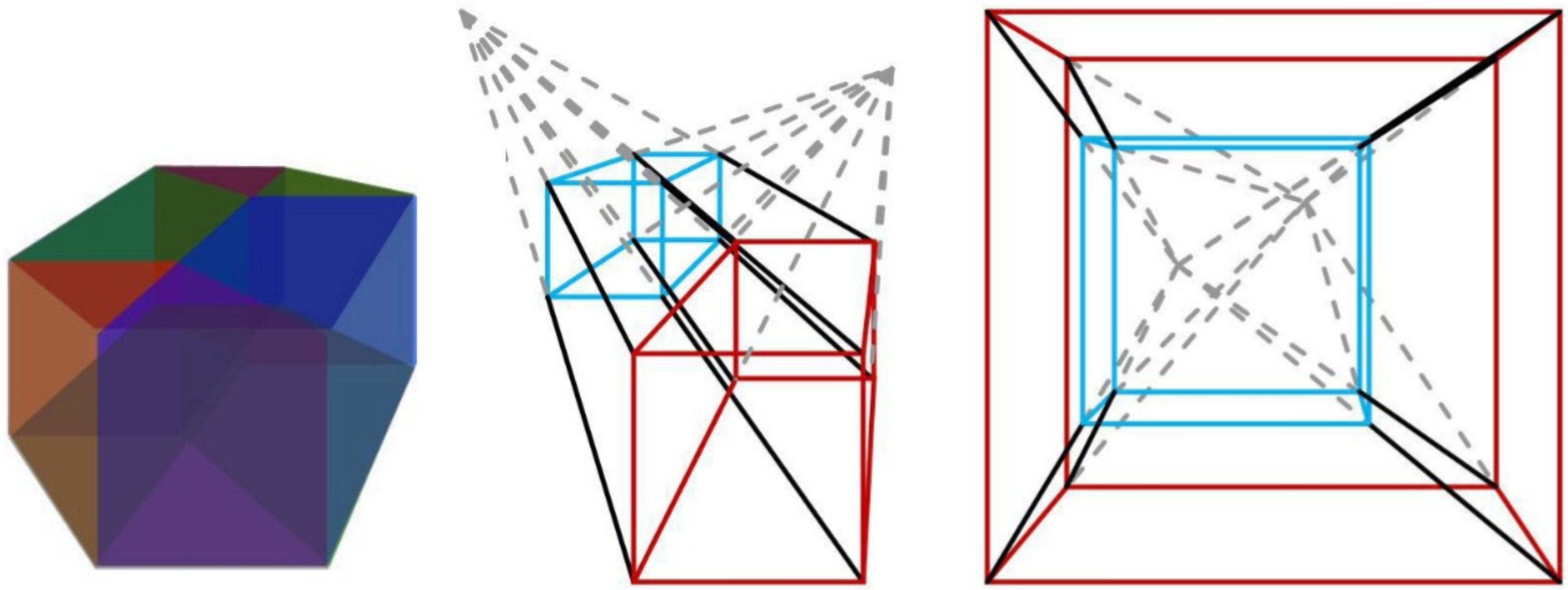
Cubes in Perspective



Tesseracts in Perspective



Tesseracts in Perspective

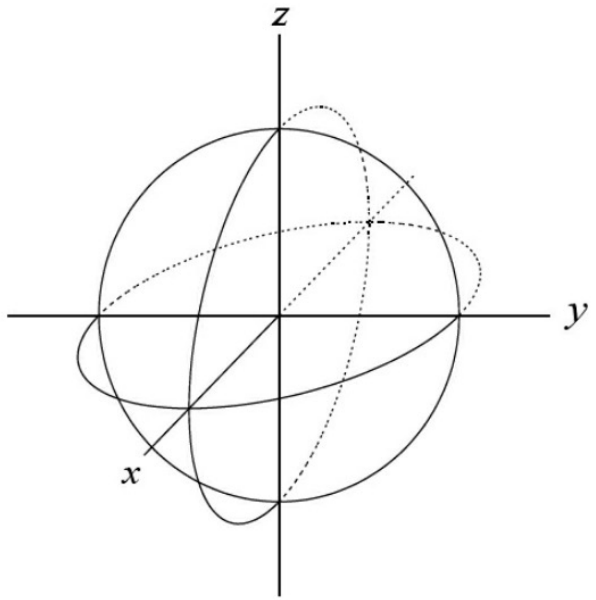


N-Dimensional Spheres

0D 1D 2D 3D

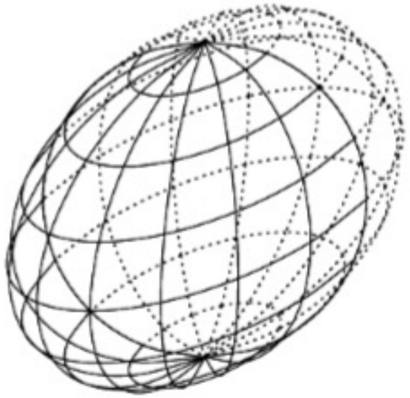
(N-1)-dimensional objects in N-dimensional space.

The Equation of a Sphere

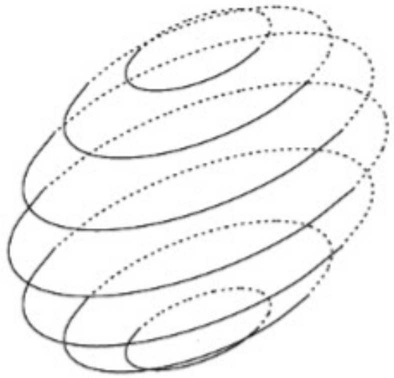


sphere $x^2 + y^2 + z^2 = R^2$

Directions of Earth



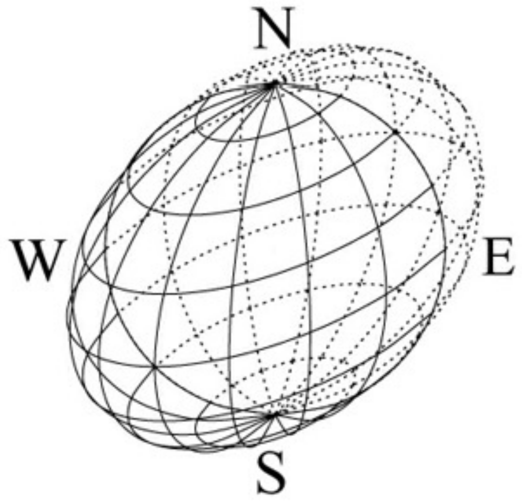
earth



latitudes

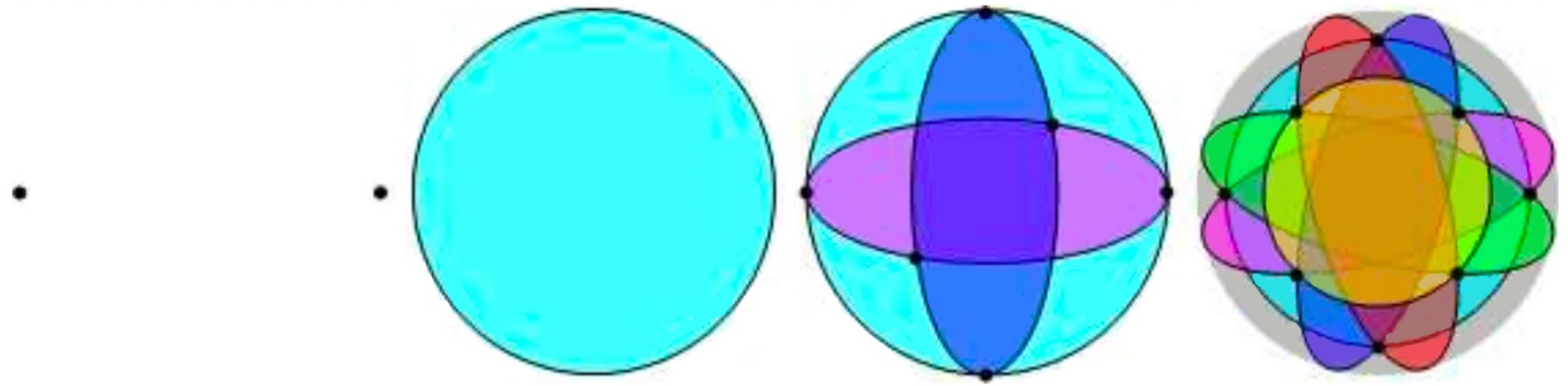
longitudes

Directions of a Sphere in 3D

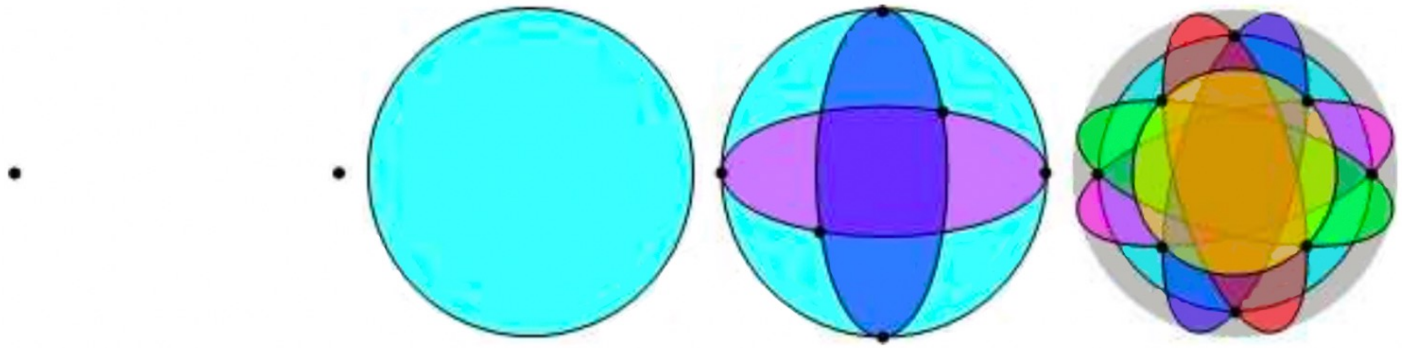


latitudes and longitudes in 3D

Hyperspheres in 4D



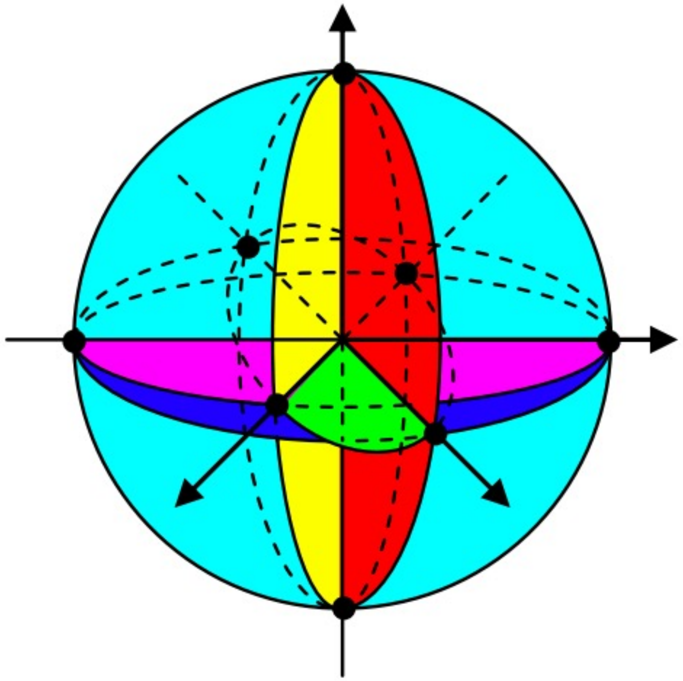
Summary



From left to right:

- In 0D space, the hypersphere does not exist. (No point can be one unit from the origin).
- In 1D space, the unit hypersphere is the set of two 0D points one unit from the origin. Its equation is $x^2 = 1$ or $x^2 = \pm 1$.
- In 2D space, the hypersphere is the 1D circumference of a circle. The equation for the unit hypersphere in 2D space is $x^2 + y^2 = 1$.
- In 3D space, the hypersphere is a 2D surface called a sphere. The equation for the unit hypersphere in 3D space is $x^2 + y^2 + z^2 = 1$.
- In 4D space, the hypersphere is a 3D hypersurface called a glome. The equation for the unit hypersphere in 4D space is $x^2 + y^2 + z^2 + w^2 = 1$.

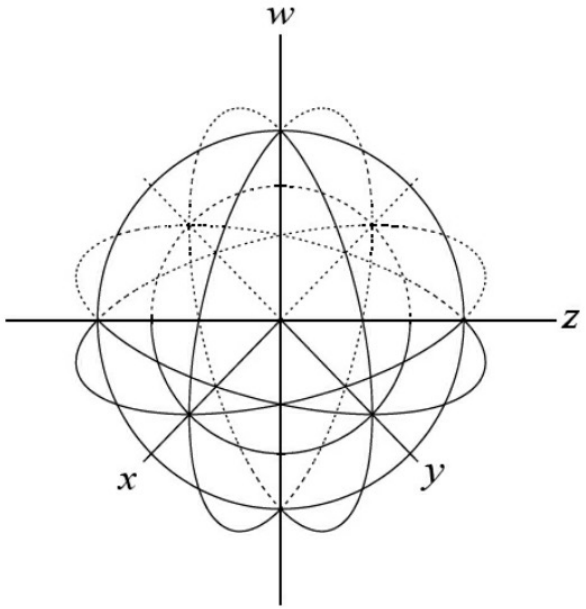
Glomes



Glomes

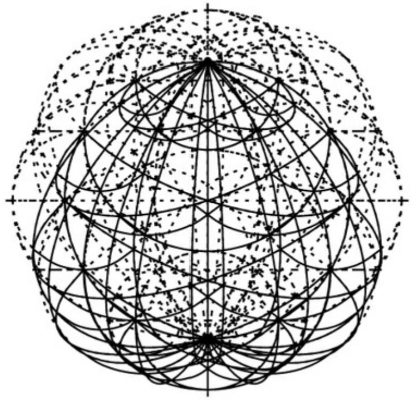
A glome is a hypersphere in 4D space.

The Equation of a Glome

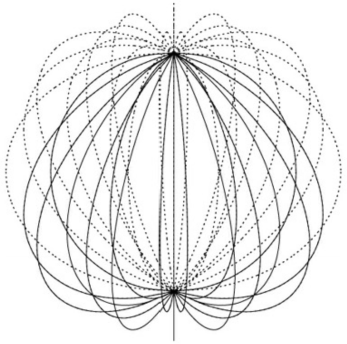


glome $x^2 + y^2 + z^2 + w^2 = R^2$

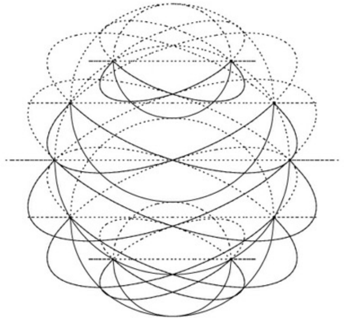
Directions of a Glome in 4D



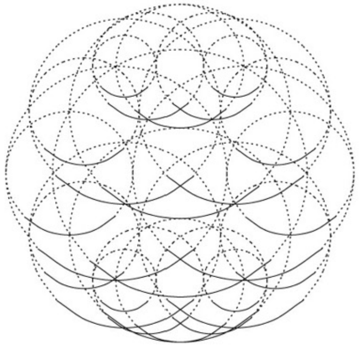
latitudes, longitudes, and hyperlatitudes in 4D



longitudes in 4D

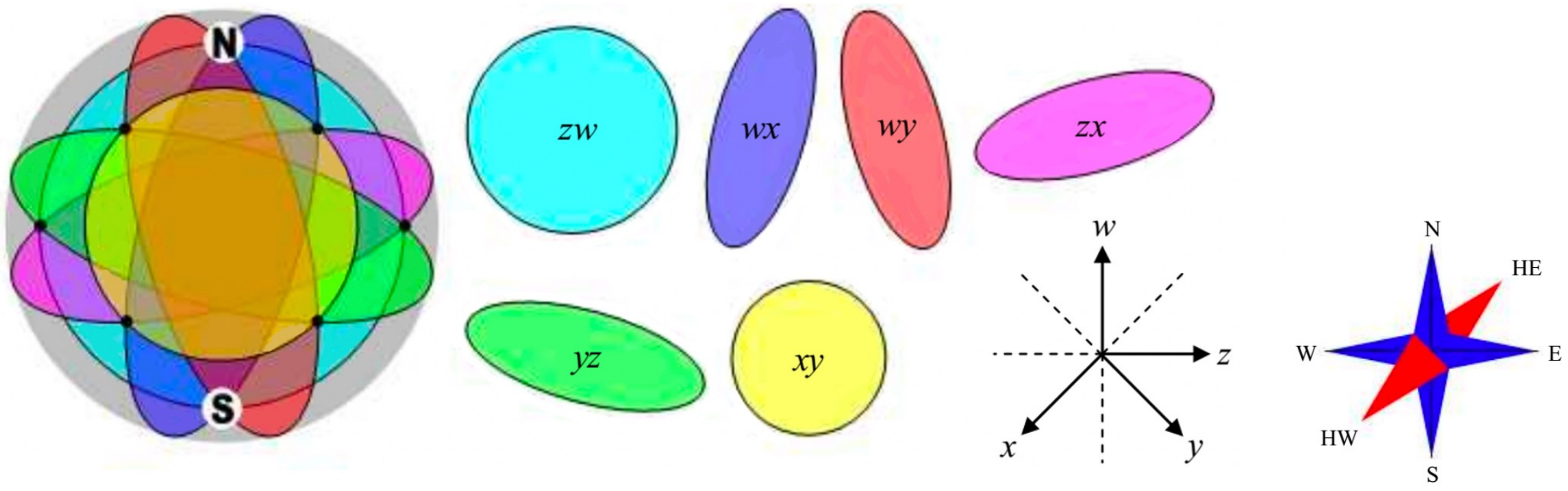


hyperlatitudes in 4D

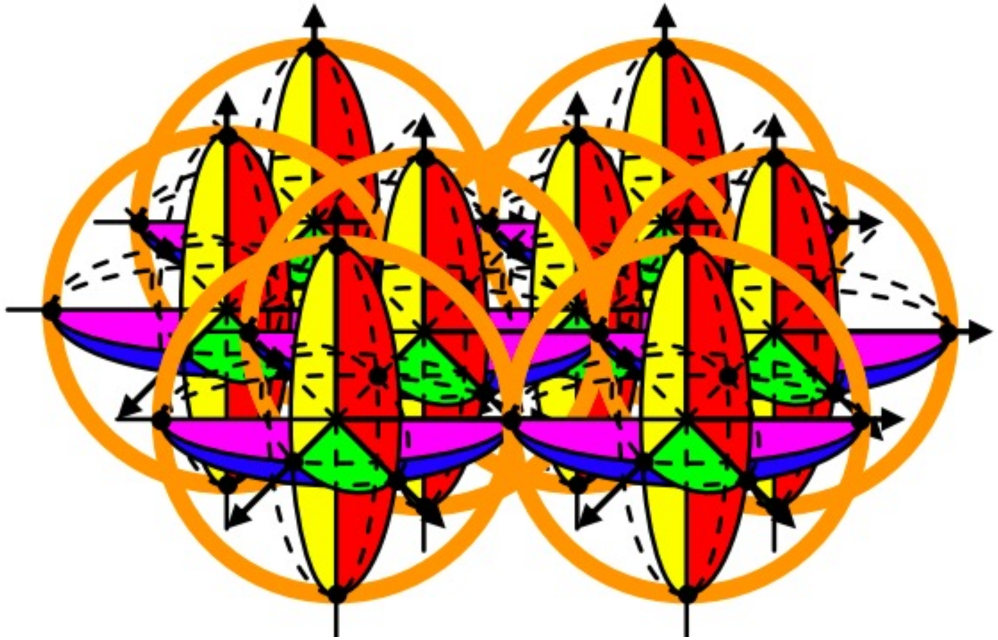


latitudes in 4D

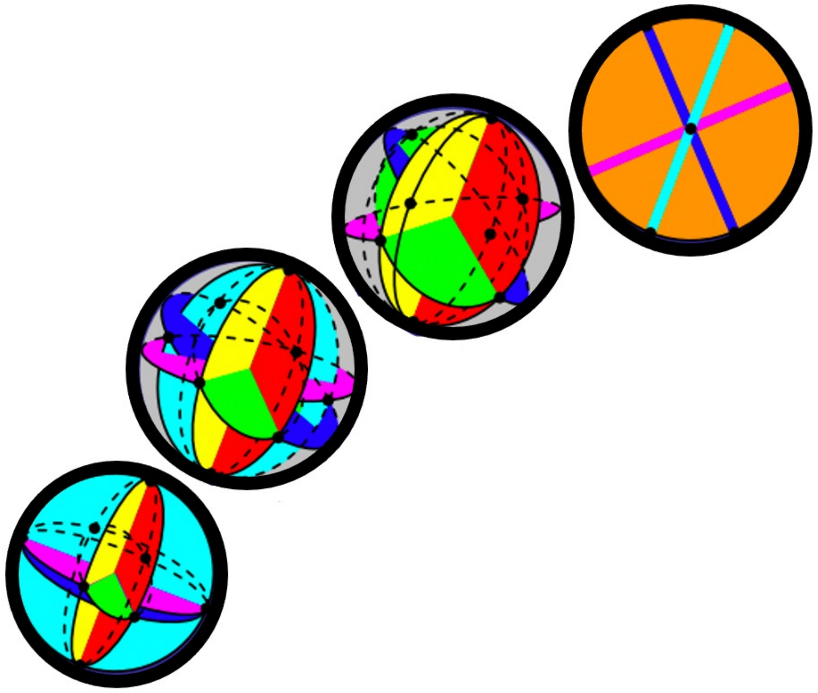
Directions of a Glome in 4D



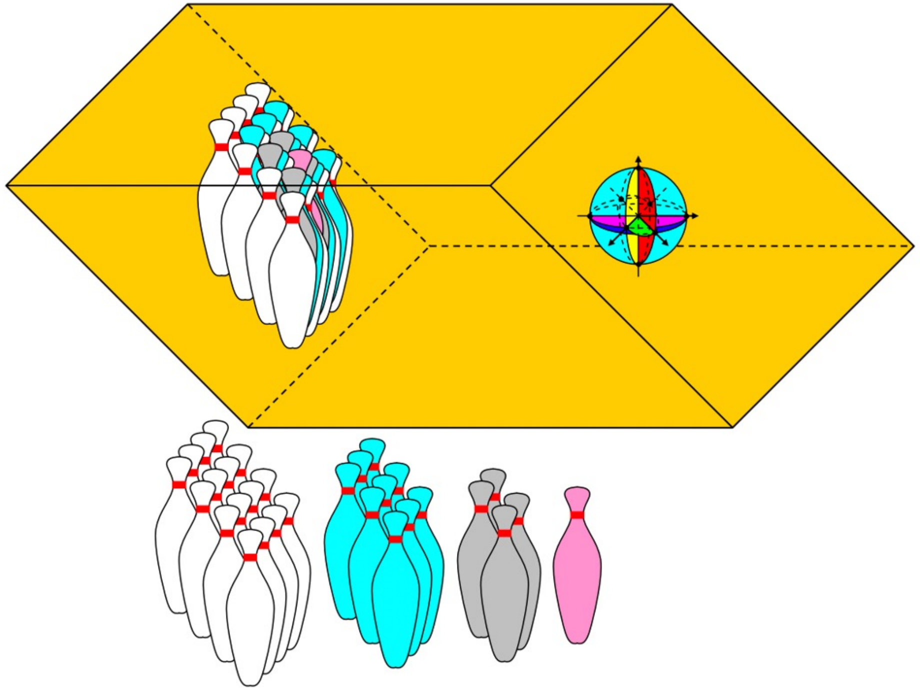
Hyperoranges



Hyperplanets orbiting around a hypersun



Hyperbowling



Polytopes

Polytopes

An N-dimensional closed shape with flat sides is called a polytope.

Polytopes

The simplest polytope is the ***polygon***, which is a closed figure in 2D with $n \geq 3$ straight edges.

Polytopes

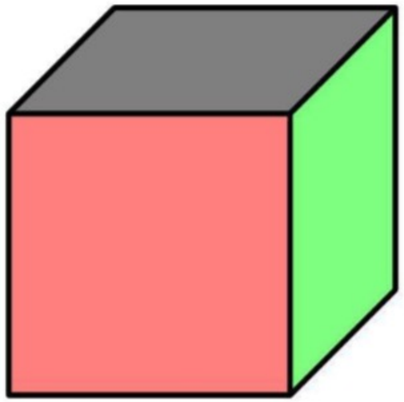
Common Polygons

Polytopes

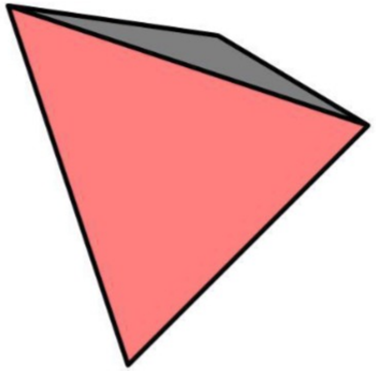
A ***polyhedron*** is a closed surface in 3D with flat sides (faces), the $n \geq 4$ faces of which are polygons.

Polytopes

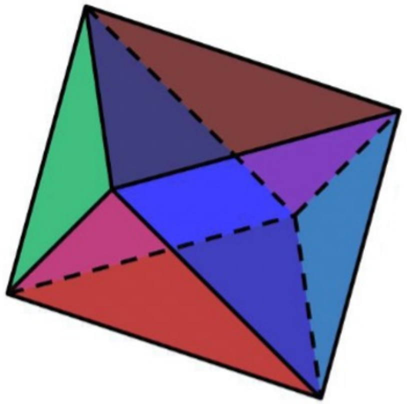
Common Polyhedra



Cube



Tetrahedron



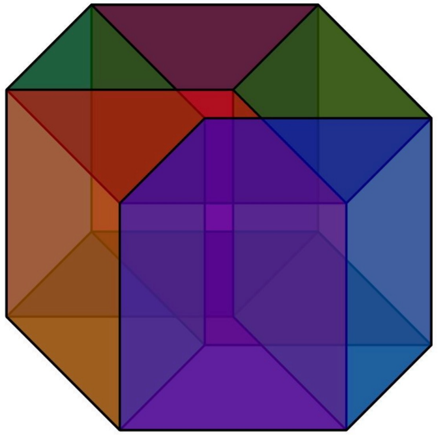
Octahedron

Polytopes

A polychoron is a closed hypersurface in 4D bounded by $n \geq 5$ polyhedra.

Polytopes

Common Polychoron



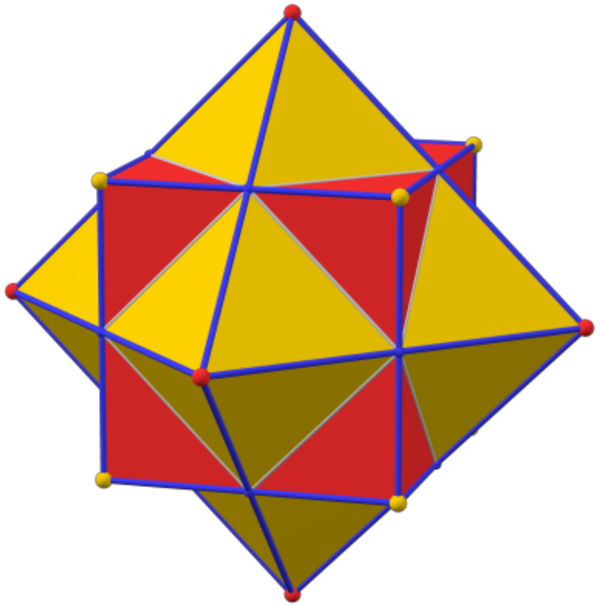
Tesseract

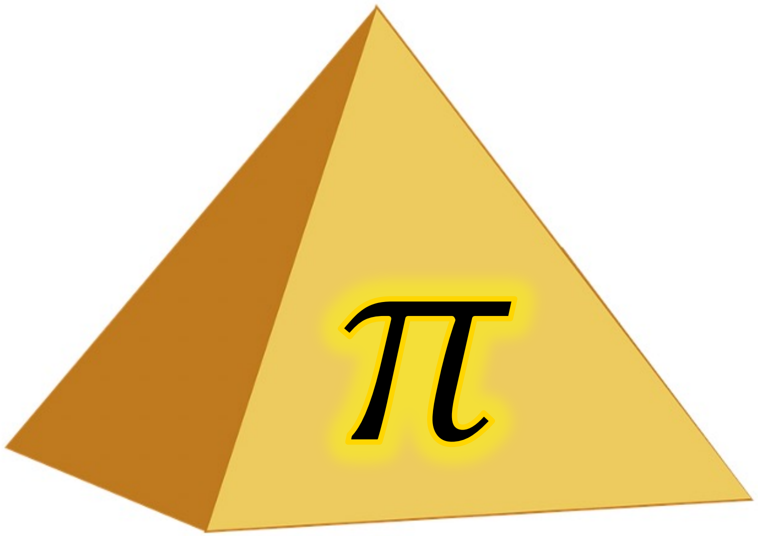
Polytopes

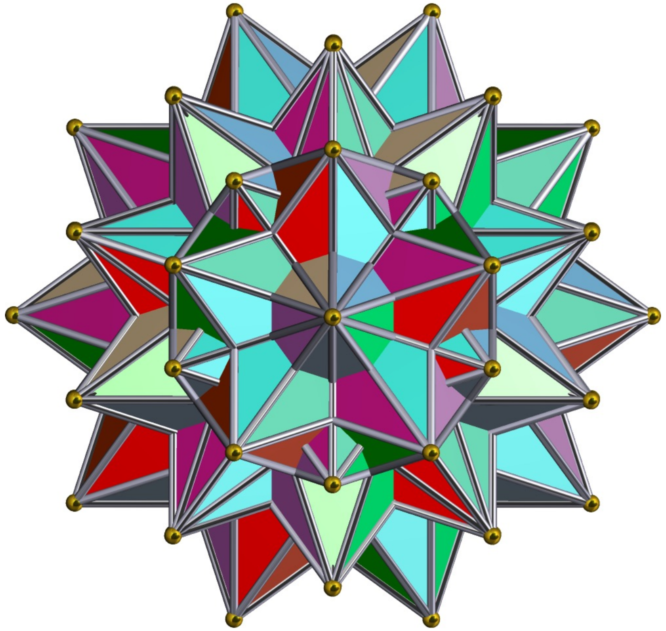
Summary:

- Polygon is a 2-polytope
- Polyhedron is a 3-polytope
- Polychoron is a 4-polytope

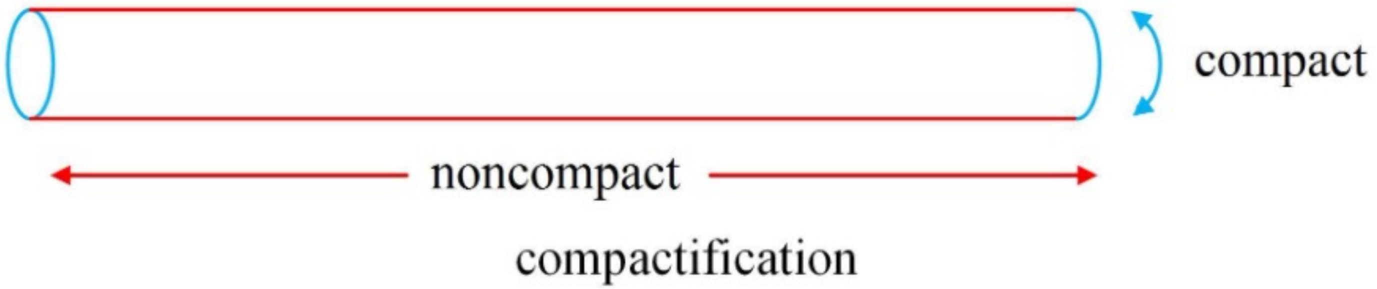
Polytopes







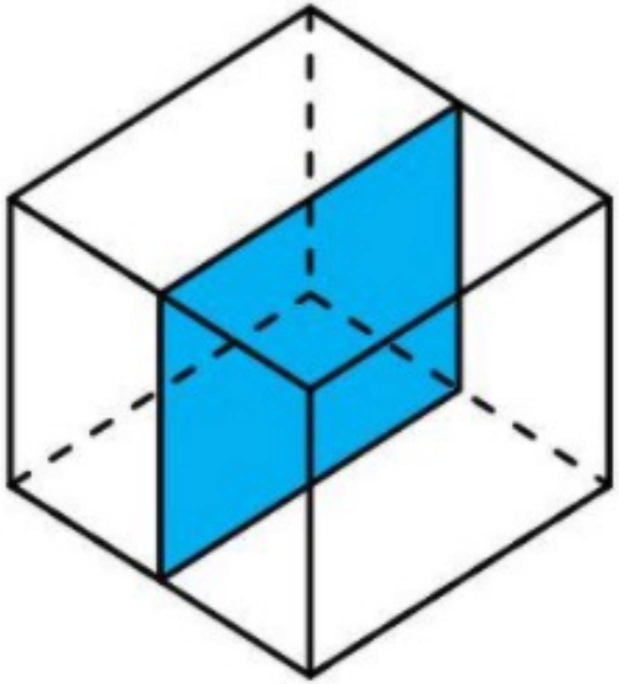
Compactification



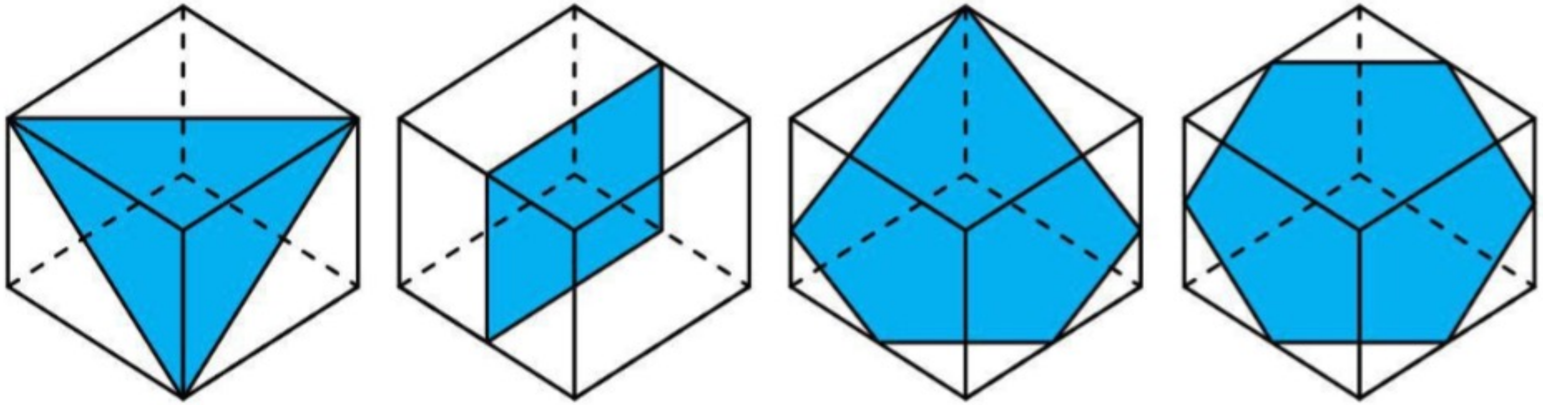
Cross Section

Imagine you have a rectangular cheesecake. If you slice the cheesecake in half, the part you sliced is considered the cross section.

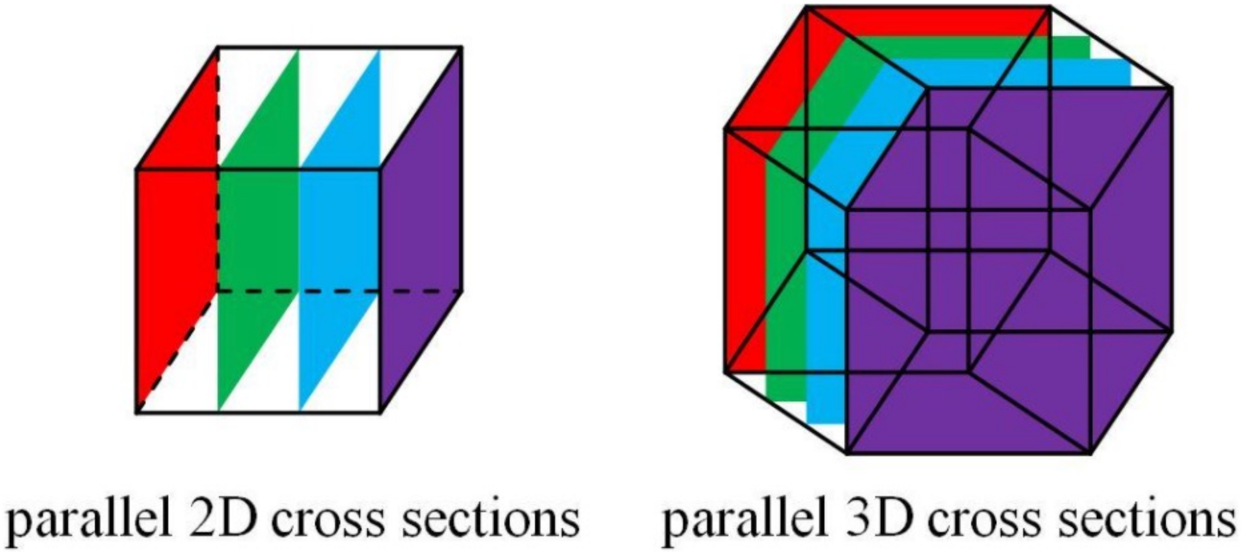
Cross Section section of a cube



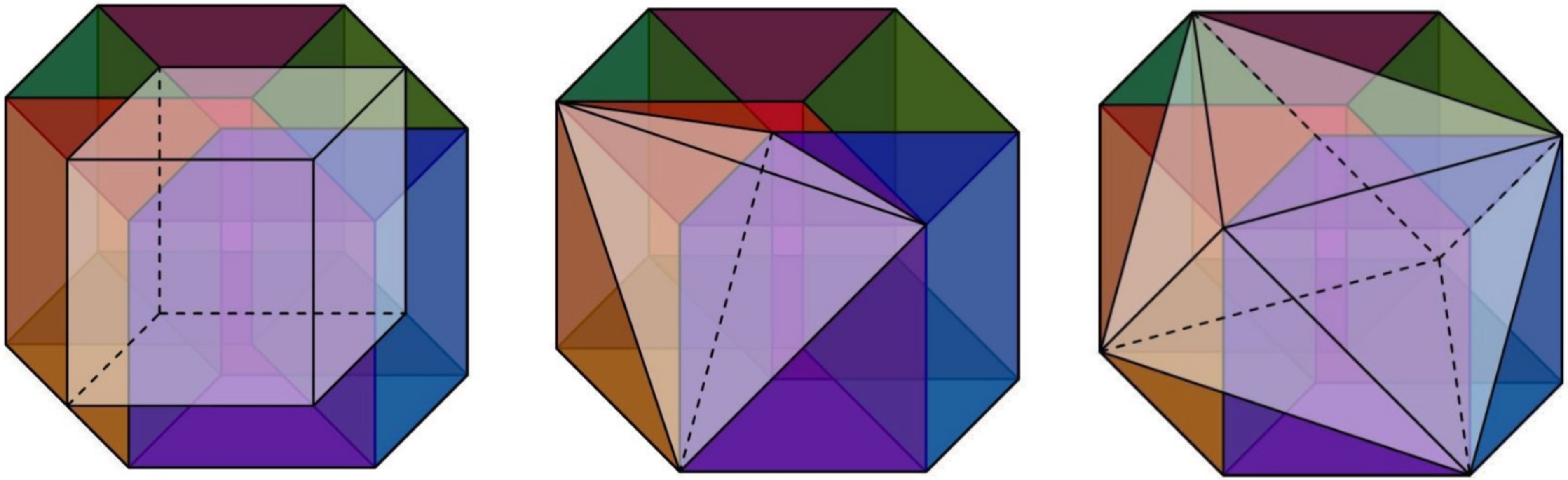
Various 2D Cross Sections of a Cube



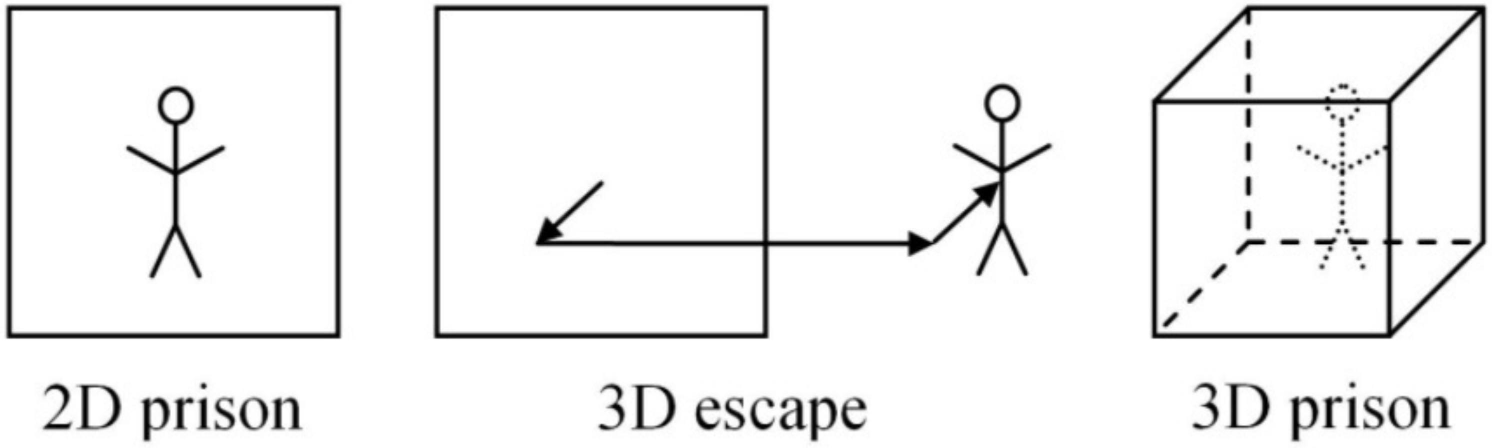
Cross sections in different dimensions



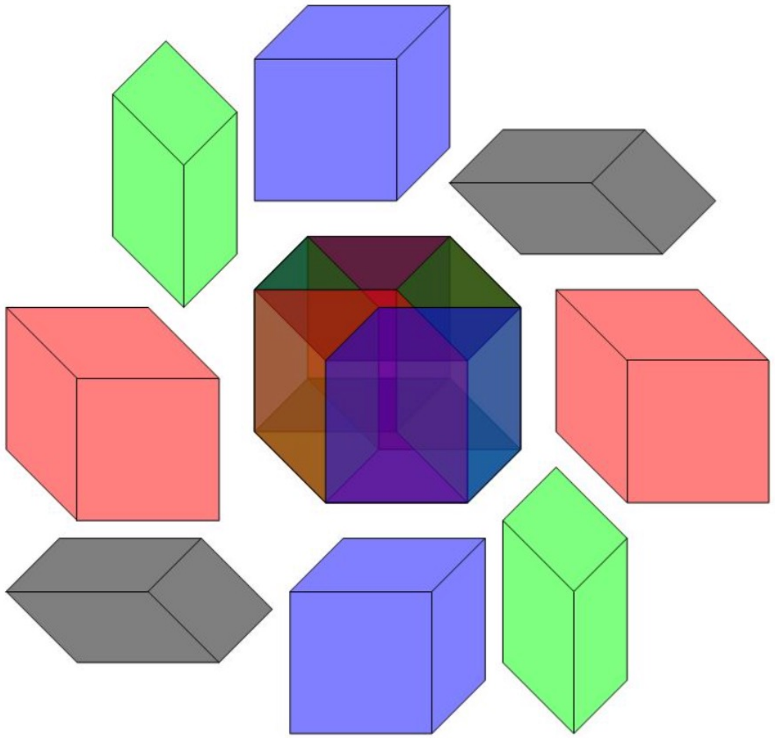
Cross Sections of a Tesseract



Escaping a 2D prison



Tesseracts are bounded by 8 cubes



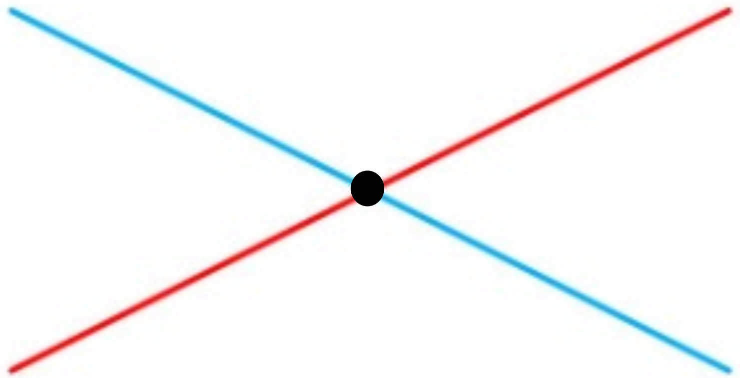
Hyperplanes

A plane is an infinitely large square.
A hyperplane is an infinitely large cube.

Hyperplanes

The 3D space we live in is considered a hyperplane.

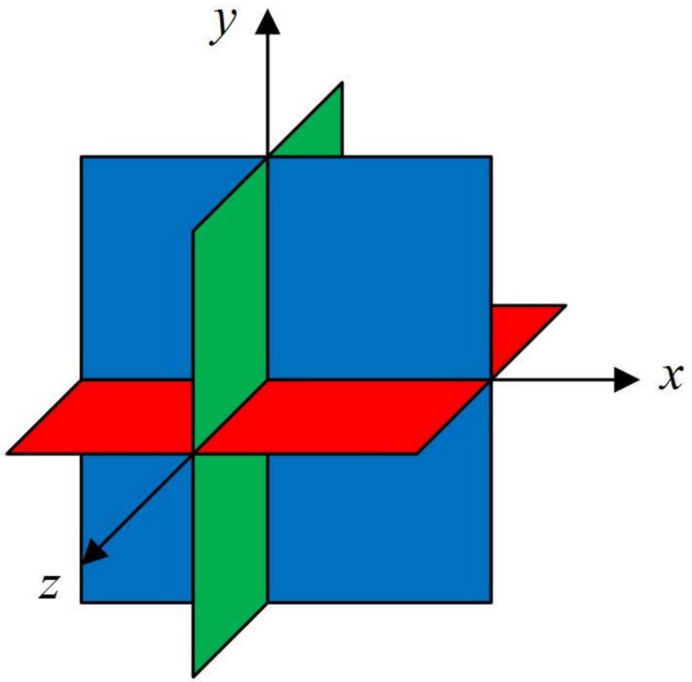
Lines Intersecting



Lines

Lines intersect at a point.

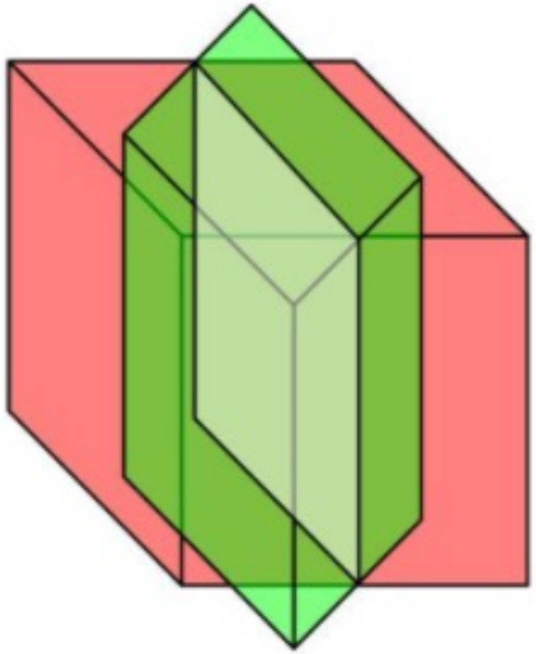
Planes Intersecting



Planes

Planes intersect at a line.

Hyperplanes Intersecting



Hyperplanes

Hyperplanes intersect at a plane.

References

[1] Chris McMullen, *The Visual Guide to Extra Dimensions: Visualizing The Fourth Dimensions, Higher-Dimensional Polytopes, and Curved Hypersurfaces*, 2014.